




Sections 2.2 (Atoms-First)
and 8.2 (Chemistry-First)
Rounding Off and Significant Figures

An Introduction to Chemistry
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Reporting Values from Measurements

- The numbers in a values should reflect both the magnitude (size) of the value and the value's uncertainty. (e.g. 185 lb)
- Measurements never give exact values. (185 ± 2 lb)
- Unless we are told otherwise, we assume that values from measurements have an uncertainty of plus or minus one in the last decimal place reported. (e.g. 184-186 lb)
 - 5 g says could be from 4 g to 6 g
 - 5.00 g says 4.99 g to 5.01 g
 - 5.000 g says 4.999 g to 5.001 g

Reporting Values from Calculations



- If a calculation is performed using all exact numbers and if the answer is not rounded off, the answer is exact, but this is a rare occurrence.
- The answers derived from calculations are usually not exact.
- As we do for measurements, unless we are told otherwise, we assume that values from calculations have an uncertainty of plus or minus one in the last decimal place reported.
- The main purpose of this lesson is to show you a simple way to round off the answers to your calculations in a way that reflects the proper degree of uncertainty.

Reporting Values from Calculations (Example)

- For example, Consider the following calculation that converts 4.9800 g to lb.

$$? \text{ lb} = 4.9800 \text{ g} \left(\frac{1 \text{ lb}}{453.6 \text{ g}} \right) = 0.01098 \text{ lb (or } 1.098 \times 10^{-2} \text{ lb)}$$

- 4.98 divided by 453.6 is 0.01097883597884, which suggests an uncertainty of 0.0000000000000001 lb.
- The value 453.6 g suggests an uncertainty of 0.1 g, which is about 0.0002 lb, so we do not want to report our answer as 0.01097883597884 lb, which suggests an uncertainty of 0.0000000000000001 lb.
- We need a simple technique for rounding our answers to calculations to reflect the uncertainty of the numbers used in the calculations.

Rounding Answers from Multiplication and Division Step 1

- **Step 1:** Determine whether each value is exact, and ignore exact values.
 - Exact values
 - Numbers that come from definitions are exact.
 - Numbers derived from counting are exact.
 - Do Step 2 for values that are not exact.
 - Values that come from measurements are never exact. (e.g. 4.9800 g)
 - We will assume that values derived from calculations are not exact unless otherwise indicated.

Exact Values

- Numbers that come from definitions are exact.
 - Numbers derived from the definitions of the metric prefixes are exact, such as

$$\frac{10^3 \text{ g}}{1 \text{ kg}}$$

- Numbers in English-English conversion factors with the same type of unit top and bottom are exact, such as

$$\frac{12 \text{ in.}}{1 \text{ ft}}$$

- The 2.54 in the following conversion factor is exact

$$\frac{2.54 \text{ cm}}{1 \text{ in.}}$$

- Numbers derived from counting are exact. $\frac{5 \text{ toes}}{1 \text{ foot}}$

Rounding Answers from Multiplication and Division Step 1 (Numbers that are not exact)

- Do Step 2 for values that are not exact.
 - Values that come from measurements are never exact.
 - We will assume that values derived from calculations are not exact unless otherwise indicated.
 - Except for 2.54 cm/1 in. (in which the 2.54 is defined and exact), we will assume that all of the English-metric conversion factors that we see have numbers that were calculated and rounded. For example, the 453.6 in the following conversion factor is not exact.

$$\frac{453.6 \text{ g}}{1 \text{ lb}}$$

Rounding Answers from Multiplication and Division Step 1 Example

- The following shows how we can convert the mass of a hydrogen atom from micrograms to pounds.

$$? \text{ lb} = 1.67 \times 10^{-18} \cancel{\mu\text{g}} \left(\frac{1 \cancel{\text{g}}}{10^6 \cancel{\mu\text{g}}} \right) \left(\frac{1 \text{ lb}}{453.6 \cancel{\text{g}}} \right)$$

- **Step 1:** Determine whether each value is exact, and ignore exact values.
 - The mass of a hydrogen atom is not defined, and masses cannot be counted, so the 1.67×10^{-18} is not exact.
 - The 10^6 number comes from the definition of micro, so it is exact. We will ignore this number when rounding.
 - Except for 2.54 cm per inch, all of the English-metric conversion factors that we will see are calculated and rounded, so the 453.6 is not exact.

Rounding Answers from Multiplication and Division Step 2

- **Step 2:** Determine the number of significant figures in each value that is not exact.
 - All non-zero digits are significant.

$\begin{array}{c} 1 \quad 3 \quad 5 \\ 11.275 \text{ g} \\ 2 \quad 4 \end{array}$ ——— Five significant figures

- Zeros between nonzero digits are significant.

A zero between nonzero digits

$\begin{array}{c} 1 \quad | \quad 3 \quad 5 \\ 10.275 \text{ g} \\ 2 \quad 4 \end{array}$ ——— Five significant figures

Rounding Answers from Multiplication and Division Step 2 (cont.)

- **Step 2:** Determine the number of significant figures in each value that is not exact.
 - Zeros to the left of nonzero digits are not significant.

Not significant figures

0.000102 kg which can be described as $1.02 \times 10^{-4} \text{ kg}$

Both have three significant figures.

Rounding Answers from Multiplication and Division Step 2 (cont.)

- **Step 2:** Determine the number of significant figures in each value that is not exact.
 - Zeros to the right of nonzero digits in numbers that include decimal points are significant.

Five significant figures — 10.200 g 20.0 mL — Three significant figures

1 3 5 1 3
2 4 2

Unnecessary for reporting size of value,
but do reflect degree of uncertainty.

Rounding Answers from Multiplication and Division Step 2 (cont.)

- Zeros to the right of nonzero digits in numbers without decimal points are ambiguous for significant figures.

220 kg

1 ?
2

Precise to $\pm 1 \text{ kg}$ or $\pm 10 \text{ kg}$?
Two or three significant figures?

Important for reporting size of value, but unclear about degree of uncertainty

$2.2 \times 10^2 \text{ kg}$

1
2

$2.20 \times 10^2 \text{ kg}$

1 3
2

Use scientific notation to remove ambiguity.

Rounding Answers from Multiplication and Division Step 2 Example

- The following shows how we can convert the mass of a hydrogen atom from micrograms to pounds.

$$? \text{ lb} = 1.67 \times 10^{-18} \cancel{\mu\text{g}} \left(\frac{1 \cancel{\text{g}}}{10^6 \cancel{\mu\text{g}}} \right) \left(\frac{1 \text{ lb}}{453.6 \cancel{\text{g}}} \right)$$

- **Step 2:** Determine the number of significant figures in each value that is not exact.
 - For 1.67×10^{-18} , the uncertainty is reflected by the 1.67, which has three nonzero digits and thus three significant figures.
 - The 453.6 has four nonzero digits and thus four significant figures.

Rounding Answers from Multiplication and Division Step 3

- **Step 3:** When multiplying and dividing, round your answer off to the same number of significant figures as the value used with the fewest significant figures.
 - If the digit to the right of the final digit you want to retain is less than 5, round down (the last digit remains the same).

26.221 rounded to three significant figures is 26.2

|
First digit dropped is less than 5

Rounding Answers from Multiplication and Division Step 3 (cont.)

- **Step 3:** When multiplying and dividing, round your answer off to the same number of significant figures as the value used with the fewest significant figures.
 - If the digit to the right of the final digit you want to retain is 5 or greater, round up (the last significant digit increases by 1).

26.272 rounded to three significant figures is 26.3

|
First digit dropped is greater than 5.

26.2529 rounded to three significant figures is 26.3

|
First digit dropped is equal to 5.

26.15 rounded to three significant figures is 26.2

|
First digit dropped is equal to 5.

Rounding Answers from Multiplication and Division Step 3 Example

- The following shows how we can convert the mass of a hydrogen atom from micrograms to pounds.

$$? \text{ lb} = 1.67 \times 10^{-18} \cancel{\mu\text{g}} \left(\frac{1 \cancel{\text{g}}}{10^6 \cancel{\mu\text{g}}} \right) \left(\frac{1 \text{ lb}}{453.6 \cancel{\text{g}}} \right)$$

- **Step 3:** When multiplying and dividing, round your answer off to the same number of significant figures as the value used with the fewest significant figures.
 - 1.67×10^{-18} has three significant figures.
 - 453.6 has four significant figures.
 - Our answer should have three significant figures.

Rounding Answers from Multiplication and Division Step 3 Example

- **Step 3:** When multiplying and dividing, round your answer off to the same number of significant figures as the value used with the fewest significant figures.
 - The calculated result is $3.681657848325 \times 10^{-27}$.
 - We round to three significant figures, and because the first number we are dropping is less than 5 (i.e. 1 in this case), we round our answer to 3.68×10^{-27} .

$$? \text{ lb} = 1.67 \times 10^{-18} \cancel{\mu\text{g}} \left(\frac{1 \cancel{\text{g}}}{10^6 \cancel{\mu\text{g}}} \right) \left(\frac{1 \text{ lb}}{453.6 \cancel{\text{g}}} \right) = 3.68 \times 10^{-27} \text{ lb}$$

Example 2.4 and 8.4: The average human body contains 5.2 L of blood. What is this volume in quarts?

$$? \text{ qt} = 5.2 \cancel{\text{L}} \left(\frac{1 \cancel{\text{gal}}}{3.785 \cancel{\text{L}}} \right) \left(\frac{4 \text{ qt}}{1 \cancel{\text{gal}}} \right) = 5.5 \text{ qt}$$

- We input the following into the calculator:

$$5.2 \div 3.785 \times 4 =$$

- The answer on the display of the calculator is 5.49537649. We do not write this down until we decide on the number of significant figures in our answer.
- 5.2 has 2 sig. figs., 3.785 has 4 sig. figs., and 4 is exact, so we round our answer to 2 significant figures.
- The first digit we are dropping is 9, so we round up to 5.5.

Example 2.5 and 8.5: How many minutes does it take an ant walking at 0.01 m/s to travel 6.0 feet across a picnic table?

$$\begin{array}{ccccccc}
 \text{2 significant figures} & & \text{Exact} & & & & \text{1 significant figure} \\
 & \swarrow & \swarrow & & & & \swarrow \\
 ? \text{ min} = 6.0 \text{ ft} & \left(\frac{12 \text{ in.}}{1 \text{ ft}} \right) & \left(\frac{2.54 \text{ cm}}{1 \text{ in.}} \right) & \left(\frac{1 \text{ m}}{10^2 \text{ cm}} \right) & \left(\frac{1 \text{ s}}{0.01 \text{ m}} \right) & \left(\frac{1 \text{ min}}{60 \text{ s}} \right) & = 3 \text{ min} \\
 & & & \swarrow & \swarrow & \swarrow & \\
 & & & \text{Exact} & \text{1 significant figure} & \text{Exact} &
 \end{array}$$

- We input the following into the calculator:

$$6 \times 12 \times 2.54 \div 100 \div .01 \div 60 =$$

- The answer on the display of the calculator is 3.048.
- 6.0 has 2 sig. figs., 0.01 has 1 sig. fig., and 12, 2.54, 10^2 , and 60 are exact, and so we round our answer to 1 significant figure.
- The first digit we are dropping is 0, so we round down to 3.

Rounding Answers from Addition and Subtraction

- **Step 1:** Determine whether each value is exact, and ignore exact values.
 - Skip exact values.
 - Do Step 2 for values that are not exact.
- **Step 2:** Determine the number of decimal positions for each value that is not exact.
- **Step 3:** Round your answer to the same number of decimal positions as the inexact value with the fewest decimal places.

Example 2.6 and 8.6

- A laboratory procedure calls for you to determine the mass of an unknown liquid. Let's suppose that you weigh a 100-mL beaker on a new electronic balance and record its mass as 52.3812 g. You then add 10 mL of the unknown liquid to the beaker and discover that the electronic balance has stopped working. You find a 30-year-old balance in a cupboard, use it to weigh the beaker of liquid, and record that mass as 60.2 g. What is the mass of the unknown liquid?

$$60.2 \text{ g beaker with liquid} - 52.3812 \text{ g beaker} = 7.8188 \text{ g liquid}$$

Example 2.6 and 8.6

60.2 g beaker with liquid – 52.3812 g beaker = 7.8188 g liquid

- 60.2 is uncertain in the tenth position (± 0.1).
- 52.3812 is uncertain in the fourth decimal position (± 0.0001).
- We round our answer off to the tenth position (7.8 g)

60.2 g beaker with liquid – 52.3812 g beaker = 7.8 g liquid